Comparison of the Theoretical Solar Radiation Effects and the Observed Accelerations of the PAGEOS Satellite

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The effects of solar radiation pressure on a balloon spacecraft, PAGEOS, are examined under the dynamical conditions in which it was observed to undergo both a speed-up in rotation and a precession in its spin-axis. The analysis is an extension of mechanisms suggested by earlier workers who had no access to the details of the spacecraft shape and dynamical orientation data, now available from ground-based photometric studies. It is shown that both spin and precession torques can be explained through the radiation pressure mechanism using the optical and thermophysical properties of aluminized mylar, and by introducting an elliposidal model for the relaxed spacecraft shape, a model which has earlier been found to explain orbital perturbations as due to radiation pressure.

Introduction

THE PAGEOS spacecraft is an aluminized, 100-ft diam plastic balloon very similar to the earlier ECHO passive communication satellites, but placed into a polar orbit with a semimajor axis of approximately 10,600 km.

Because of their large areas and small masses, spacecraft of the PAGEOS, ECHO, and Air-Density Explorer types had been expected to lose quickly any residual rotation following their inflation. In fact, several mechanisms for momentum loss have been proposed, e.g., eddy current and drag friction. However, observation shows that all of the shell-type spacecraft retain or acquire appreciable rotation persisiting after five or more years in orbit.

PAGEOS, orbiting well above the atmosphere of the earth and protected from the direct solar wind by the geomagnetic cavity, has in contrast exhibited some interesting but unexplained dynamical behavior since shortly after its launch in 1966. Its rate of rotation is observed to be highly variable.[1,2] When part of the orbit passes through the shadow of the earth, PAGEOS tends to decrease in rotation rate. At a later time when the orbit is again totally in sunlight, PAGEOS then increases its rotation rate. Recent analysis of observations,[1] which demonstrate these effects on the balloon spin rate, also show that the satellite spin-axis precesses about a particular direction in space, namely, the direction from the spacecraft to the sun, at an essentially constant precession angle of 65°, but with a varying rate of precession also dependent on the orbit sunlit/shadow condition.

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Index categories: Spacecraft Attitude Dynamics and Control; Thermal Modeling and Experimental Thermal Simulation; Spacecraft Configurational and Structural Design (including Loads).

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The correlation between the observed motions and the direction to the sun suggests strongly that the torquing forces present on the body are induced by solar radiation pressure. We examine the radiation pressure interactions with the PAGEOS spacecraft based on a model which includes elliposidal deformations to the spacecraft, based on optical photometric measurements, as well as the optical and heat transfer properties of the spacecraft structural materials, and will show that radiation pressure is the probable cause of the spin-axis precession. These ellipsoidal deformations must be mechanical in origin and are much larger than the purely thermal ones suggested by earlier works to explain spin acceleration. A thermally distorted sphere is shown to have much too small a precession torque and in the wrong sense.

Previous Studies

The extent to which solar radiation pressure can explain perturbations to the orbital motion, especially the orbital eccentricity, has long been realized.[3,4] Its influence on other orbital parameters of distorted balloon satellites, such as mean anomaly, has recently been shown in an approximate theory by Smith and Kissell,[5] which incorporates an ellipsoidal model for the shape of the spacecraft. The success of the radiation-pressure mechanism in explaining the complex orbital perturbations, along with the failure to explain the rotational perturbations by other mechanisms,[6,7] argues for a more detailed look at radiation pressure as the torquing mechanism for spin acceleration and as a possible cause for the precessional torque.

Mar and Vigneron[6] were the first to propose that a spin-accelerating torque would be produced on ECHO-type satellites by the effects of solar radiation. They suggested that the body of the satellite would be unevenly heated and thus thermally distorted from a purely spherical form. Radiation pressure on the resulting asymmetric body would then produce the required torque. By arguing that only radiation transport, internally and externally, determines the temperature distribution on the rotating shell, they predicted a slight area asymmetry for the spacecraft.

This asymmetry of spacecraft area with respect to the center-of-mass predicted reasonable torques for ECHO II, but a numerical integration of the torque to predict the rotation history diverged from the observed rotation rates after a few months. Their study was limited to consideration of a thin-shell spacecraft whose spin axis was oriented 90° to the solar vector, a reasonable model if the asymmetric distortion was due entirely to thermal expansion. This simplification, however, rendered the temperature distribution expressions of Mar and Vigneron incomplete for this study.

Slabinski[7] re-examined the problem of thin-shell spacecraft in search of other mechanisms for acceleration torques. He derived complete expressions for the temperature distribution which allowed for a varying angle between the solar vector and the spin axis. His temperature relationships were used in this study.

This present study will confine itself to that period of time covered in August-December 1969, when Vanderburgh and Kissell were able to define the rotation rate of PAGEOS, the rotation-axis location, and by differentiation, the spin acceleration and precession rates for the spacecraft rotation axis. Figure 1 is taken after Fig. 8, Ref. 1, and indicates that during November 1969, PAGEOS experienced a maximum angular acceleration of $\dot\omega=0.72\times10^{-8}$ rad/sec^2. Simultaneously a precession rate was observed of 47°/day. This reaction occurred when the spin period was 240 sec. We seek to show that solar radiation pressure, acting upon a reasonable model of the spacecraft optical properties and mechanical form, will be sufficient to explain both the rotational and precessional motions of the spacecraft.

Reference Frame and Apparent Torques

Further explanation will be aided by establishment of a set of Cartesian coordinate axes referenced to the spin axis and the solar vector. The z-axis in the plane formed by the z-axis and a vector to the sun. The y-axis is defined such that it completes an orthogonal right-hand set with the x- and z-axes. The coordinate axis is depicted in Fig. 2, which also designates regions of the spacecraft which undergo different conditions of solar illumination as the result of spacecraft rotation.

The moment of inertia of PAGEOS will be approximated by assuming it to be a homogeneous spherical shell.

$$I=2/3\ M\ R_o{}^2=8380\ \text{kg m}^2 \tag{1}$$

The observed dynamics of PAGEOS[1] imply that a torque-T_z must exist about the z-axis to change the spin rate ω.

$$T_z=I.\dot\omega=6\times10^{-5}\ \text{Nm} \tag{2}$$

A torque must also exist about the y-axis to cause the observed precession of $\omega_p=47°/\text{day}=9.5\times10^{-6}$ rad/sec°.

$$T_y=I\omega\ \omega_p\sin(\psi)=2\times10^{-3}\ \text{Nm} \tag{3}$$

In Ref. 1 the angle between the solar vector and the spin vector was found to be a constant 65°, and it was established that the south pole of PAGEOS was in constant sunlight if we use the right-hand rule to define the sense of rotation analogous to the earth. In the developments which follow, the solar vector is placed in the northern hemisphere to simplify the derivations. We must specify ψ as the sun vector-spin axis angle from the north rotational pole. In the PAGEOS case $\psi=180°\text{-}65°=115°$. The precessional torque given in Eq. (3) is then algebraically consistent with the coordinate frame and will be the basis for validation of theoretical torques determined by this study.

Temperature Distribution on the Spacecraft

The development of the temperature distribution equation of Slabinski will not be reproduced here; the assumptions, approach, and results will be summarized only (a complete treatment will be found on pp. 249-258 of Ref. 7).

The seven basic assumptions of Slabinski are: 1) The total solar flux absorbed by the spacecraft is determined by its projected area, its external absorptivity α, and the solar constant E_s. 2) The total energy radiated by the spacecraft may be expressed as a product of its total surface area, the Stefan-Boltzman constant σ, and the fourth power of some mean temperature $\bar T$, and the outer emissivity ϵ_o of the external surface. The local temperature T will vary about this level depending upon the direct and duration of exposure to direct sunlight. He assumes a representation

$$T=T(1+\gamma+\gamma^2/2+\ldots) \tag{4}$$

3) $\alpha=0.109$ and $\epsilon_0=0.03$ and are assumed constant. 4) Earth-albedo flux is assumed insignificant. 5) The emissivity ϵ_i of the internal surface is assumed uniform and constant at a value of 0.45. 6) The energy density of a hollow sphere is homogeneous. This leads to a ratio of the energy emitted by each area element toward the interior to that emitted toward the outside as being in the ratio ϵ_i/ϵ_0. 7) The surface shell is of uniform area density ρ kg/m^2 and specific heat C_p ergs/kg-K.

Based on the previous assumptions and defining the mechanical axis of rotation relative to the solar direction and the rotation rate (ω rad/sec), a differential equation can be established relating the radiation gain or loss rate from a surface element of area dA as a function of the direction of the surface normal $\bar n$ and the solar direction $\bar s$.

The resulting relationship developed by Slabinski is the first-order, nonhomogeneous partial differential equation.

$$C_p\rho\ \omega\ \bar T(\partial\gamma/\partial\phi)=E_s\alpha[\cos^+(\beta)+(15/4)] - 16\ \epsilon_0\bar T^4(1+4\gamma) \tag{5}$$

to the first-order of the binomial expansion in the parameter γ. $\cos^+(\beta)$ is a rectified function given by $\cos^+(\beta)=\bar n\cdot\bar s$ for $\pi/2$, but $\cos^+(\beta)=0$, $\beta>\pi/2$.

One finds four separate solutions to this relationship, solutions which must be continuous at the boundaries of the four distinct regions of the spacecraft. These regions are the continuously-illuminated polar cap, the continuously-dark polar cap, the bright-side (solar-illuminated) region, and the dark-side region. In general the solutions are obtained by the methods of complementary function/particular integral. The general solutions will be found in Slabinski. The dark cap is found to have a constant temperature determined by the ratio

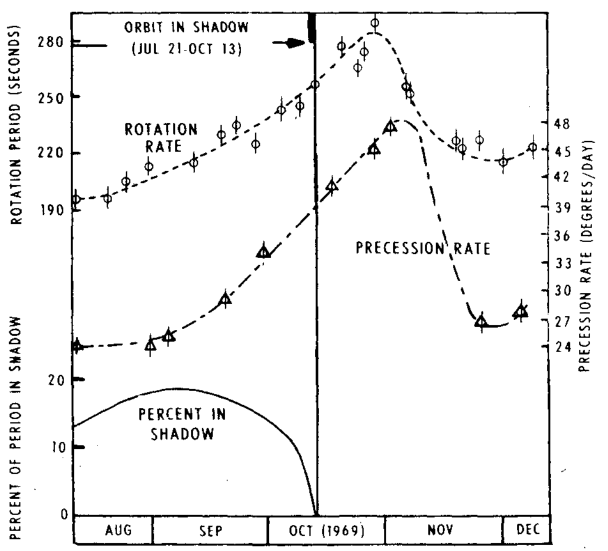

Fig. 1 Observed dependency between spin-axis precession rate and rotation period.

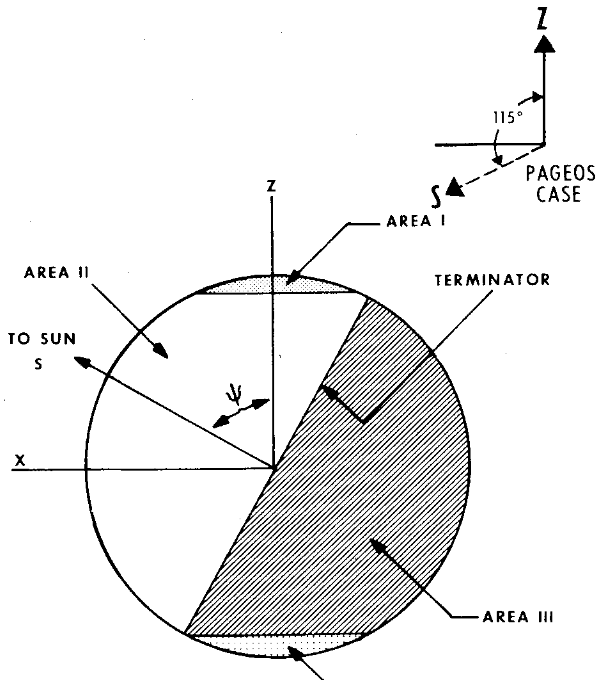

Fig. 2 Side view of PAGEOS showing solar vector and areas.

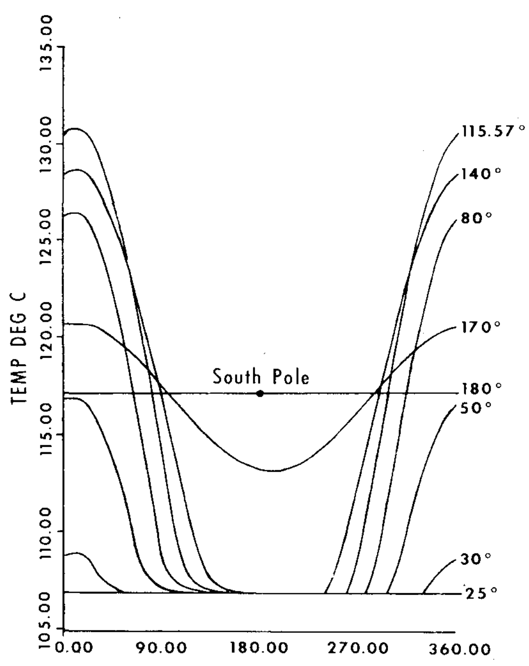

Fig. 3 Temperature profiles at selected co-latitudes for the PAGEOS spacecraft with a constant precession angle of 115° and a spin period of 240 sec.

of internal to external emissivity. the other regions are either periodic or periodic/transcendental in their temperature variations. The temperature distributions for the four regions were solved for the PAGEOS spacecraft under the assumption that the precession angle is constant at 115°. A plot of the temperature at selected co-latitudes is shown in Fig. 3.

Thermal Distortion from a True Sphere

The displacement in radius of the heated sphere from the mean radius was found to be a linear function of the temperature change, following Mar and Vigneron,[6]

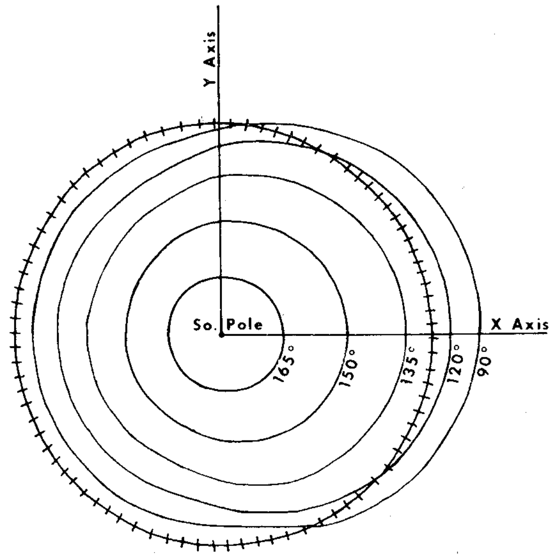

Fig. 4 Polar view of distorted PAGEOS with selected contours. The cross-hatch circle is an undisturbed equatorial contour.

$$R = R_0(1 + C_{l\Delta}T) \tag{6}$$

where C_l is a thermal expansion constant of the skin material.

Figure 4 is a south polar view of the distorted PAGEOS, with contour lines at 165°, 150°, 135°, 120°, and 90° co-latitude. The thermal distortion from a circular cross section is magnified in the figure by a factor of 100 for visibility. The cross-hatched circle represents the undistorted equatorial form.

Thermal Shift in Vehicle Center-of-Mass

As a result of the thermal expansion of the skin material, small shifts of the surface mass elements occur which result in a relocation of the center-of-mass from that of an isothermal sphere. The shift is computed by determining the moments about the X, Y, and Z axes, summing the effects of thermal expansion in each of the four regions previously discussed. We have used a numerical integration technique using 1°×1° elements of the surface to evaluate the three moments over each of the four regions and to sum the final shift of the mass elements, and find, for the 30.48 m diam sphere, $\Delta X = 0.7755$ cm, $\Delta Y = 0.1252$ cm, $\Delta Z = 0.3424$ cm.

Solar-Radiation Pressure

Radiation pressure affects each surface element in two ways, depending upon 1) a specular interaction or 2) a diffuse-scatter or absorption interaction with the surface. The specularly scattered radiation produces a force normal to the surface F_{sp} which is directed more or less toward the center of the spacecraft. In addition, the radiation absorbed or subsequently diffusely scattered produces a force F_s directed along the antisolar vector.

$$F_{sp} = 2\,R_{sp}P_0\cos^2(\beta)\,\mathrm{d}A \tag{7}$$

$$F_s = (1 - R_{sp})P_0\cos(\beta)\,\mathrm{d}A = (\alpha + r_{\text{diff}})P_0\cos(\beta)\,\mathrm{d}a \tag{8}$$

Here R_{sp} is the specular reflectance, R_{diff} the diffuse reflectance, α the absorptance, P_0 the momentum of sunlight at earth's orbit, β the angle between the surface normal and the solar vector, and dA is an element of area.

These relationships neglect the force caused by diffusely-scattered light itself since the diffuse reflectance is quite small (0.022). This resultant force would be directed very nearly along the surface normal, on the basis of symmetry arguments, and must be compared with $2R_{sp}$, more than two orders of magnitude larger. Other forces induced by the surface thermal emittance and inner radiative flux were also neglected due to the near-symmetry of the body and the relatively isothermal nature of the surface.

Determination of Radiation-Pressure Torques

Numerical integration techniques were used to compute the torques produced about the center-of-mass by the previous forces on the surface elements exposed to the solar radiation. These elemental torques were resolved and summed about the Y' and Z' axes of the distorted sphere where the primed axis refer to the Cartesian set shifted to the new center-of-mass. Surface elements of $1° \times 1°$ were used, and the integration led to $T_{Y'} = -2.3 \times 10^{-6}$ Nm; $T_{Z'} = 1.9 \times 10^{-5}$ Nm. A torque causing spin accelerations on PAGEOS must act about the z-axis. The theoretical torque resulting from thermal distortions of a simple sphere was found to be 1.9×10^{-5} Nm while the observed maximum torque was 6×10^{-5} Nm.

The deficiency of a factor of three is believed not enough to invalidate solar radiation as the prime cause of spin acceleration. The prediction of a torque of the same order of magnitude, coupled with the knowledge that the spin rate does increase when PAGEOS moves into constant sunlight, lends credence to the theory that solar pressure, acting on the thermally-distorted spherical body, can be the cause of a driving torque. As will be developed following, it is almost certain we are dealing with a more complex, nonspherical body in any case.

Thermal Expansion Effects on Spin Rate

In the case of tumbling cylindrical spacecraft and of spin-stabilized vehicles with mass distributions near the extremities, changes in spin-rate due to cooling while the earth's shadow have been noted.[8] These are a direct consequence of conservation of angular momentum while the vehicle undergoes a small dimensional change. For the PAGEOS spacecraft, a similar effect should occur; however, it can be easily shown that its effect will not persist for more than one rotational period of the spacecraft, i.e., 3-5 min. This can be seen from the thermal response time of a surface element of the spacecraft to the solar flux after emergence from the earth's shadow.

Consider a 1-m^2 element of the surface at the extreme assumption of absolute zero temperature. This element has a mass of 18.54 gm and a heat capacity[8] of 38.94 joules/°C. With an absorptivity of 0.109, it receives 155 joules/sec from the solar radiation. This yields a temperature rise of approximately $\Delta T = 155/38.9 = 4.0$ °C/sec. To reach the mean spacecraft temperature of 118°C will require only $\Delta T = (118 + 253)/4.0 = 371/4.0 = 93$ sec. Thus all of the photometric data on which the dynamical periods were measured were taken with the spacecraft at equilibrium temperature and no spin-rate effects due to thermal expansion should have been detectable. The spacecraft accommodates more rapidly than the spin period can be inferred from scattered sunlight.

Discussion of Precession Torque

The calculated torque about the y-axis of a simple sphere distorted by the Mar-Vigeron mechanism was found to be 2.3×10^{-6} Nm while the torque deduced from photometric data was 2×10^{-3} Nm. Thus the torque predicted by the model is both in the wrong sense and too small by three orders of magnitude. This result casts serious doubt on the Mar-Vigneron/Slabinski model.

The original proposition was that the primary distortion to the spherical body must come from uneven heating of a rotating sphere, and that the force must be solely because of momentum exchange between the surface and solar electromagnetic radiation. We recall that Smith and Kissell[5] found indirect support for the prolate spherical form suggested by Vanderburgh and Kissell,[1] this support coming from a remarkable agreement with complex orbital period perturbations which could not be explained by an isotropic scatter (spherical spacecraft).

If a separate mechanical distortion is already present in the figure of the spacecraft, a distortion much greater than the thermal distortion previously predicted, then solar momentum exchange might still be the primary source of the precessing torque if the basic mechanical form has a strong influence on the torque produced about the y-axis.

Precession Torque on a Rotating Prolate Spheroid

Thus we will abandon the assumption that PAGEOS can be approximated by a thermally-distorted sphere. The optical observations by Vanderburgh and Kissell indicate the actual shape approximates a prolate spheriod with ratio of major to minor axes of 6:5.

Our proposed replacement for the form of the spacecraft is a prolate spheroid of axes 6:5:5. This spheroid is in rotation about a minor axis at a rate of one rotation per 190-280 sec. We observe that this mechanical deviation from a simple sphere is much greater than the Mar-Vigneron thermal distortion. If the prolate spheroid were symmetric and in an initial state of rotation, the Mar-Vigneron mechanism would be present, but the details of the actual form would probably depart from a mathematical spheroid and make it unproductive for a detailed calculation of the z-axis torque because of the thermal distortion mechanism. We propose that the mean value of the Mar-Vigneron torque on a thermally-distorted slightly-prolate spheroid will not depart markedly from that acting on a thermally-distorted sphere of the same general size, and will not be less in value.

While we postulate the spacecraft to be a thermally-distorted prolate spheroid, we seek to simplify the calculation of the precession torque, T_y. Fortunately, the precession torque about the y-axis is one which operates on a long-time basis, being effective over a period of days, so that the details of the rotation-by-rotation scattering are much less important than their time average over 50-100 rotations, i.e., 150-400 min. thus it is the time-averaged profile which will determine the direction and magnitude of the mean precession torque.

During each rotation about a minor axis, this prolate PAGEOS will sweep out the volume of an oblate spheroid with the ratio of major to minor axes of 6:6:5. In contrast to the very difficult analytical problem of computing the torques acting on the time-varying profile of the adopted prolate spheroid model, we tackle a constant-solar-aspect, mean oblate spheroid which should be equivalent insofar as T_y is concerned. This allows use of a simple analytic expression for a constant-solar-aspect, mean, oblate spheroid: $R = R_0 [0.917 + 0.183 \sin(\theta)]$.

The torque about the y-axis for this mean oblate spheroid, of equatorial to polar-diameter ratio 6:5, was found using the same geometrical and integration methods employed for the distorted sphere. This yields a value of $T_Y = 2.5 \times 10^{-3}$ Nm compared to the observed torque of 2×10^{-3} Nm. This is a very satisfying result in that, in both *direction* and magnitude, it admits solar-radiation momentum exchange to the the cause of the continuous precession of the vehicle spin axis about the sun. The complete derivation of temperature, deformation, and torque relationships can be found in Ref. 9.

Conclusions

Evidence is mounting that solar-radiation momentum exchange is an important factor in the orbital, and the body dynamics of spacecraft, with either large area-to-mass ratios or with significant asymmetries in form. Use of the mechanical deformations of spacecraft shape obtained from ground optical observations substantiates that precession effects discovered present in thin-shell spacecraft can be produced by solar radiation pressure. Purely thermal distortions of thin-shell spacecraft, predicted earlier, cannot explain these dynamic effects or distortions, yielding torques 3 orders of magnitude too small to explain the observed precessions.

The degree of importance of these results to body dynamics is largely determined by the presence of some regular rotational motion, so that the nonisotropic optical properties allow integration of the very-low level forces or torques into a significant perturbation. These torque effects must also be considered for more compact spacecraft if stabilization levels of a few arc-minutes are to be obtained. They can be put to useful advantage in providing spin-propulsion, as has already been the case in the Canadian Alouette II,[9-11] and in precession-scanning propulsion where the solar vector might be a useful reference. It is suggested that some future balloon-type spacecraft be instrumented with skin-temperature sensors so that the radiative transports models discussed here may be validated.

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